Chapter 4

1

Motion in two dimensions

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Lecture 02

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Projectile Motion

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Projectile Motion

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- The motion of an object in both the x and y directions simultaneously, under the influence of gravity alone.
- It is a motion in two dimensions

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Assumptions of Projectile Motion

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The free-fall acceleration is constant over the range of motion.

- It is directed downward.
- $\vec{a} = \vec{g} = -9.8\hat{\jmath}\ m/s^2$

The effect of air friction is negligible.

With these assumptions, an object in projectile motion will follow a parabolic path.

- This path is called the trajectory.

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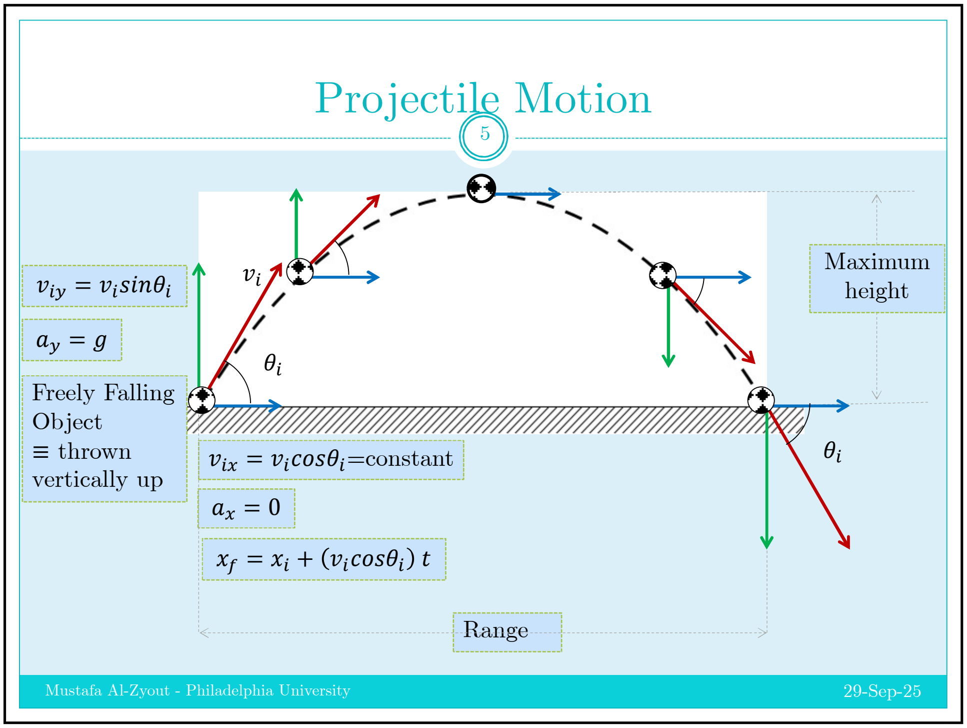
Projectile Motion
5
$v_{iy} = v_i sin\theta_i$
$a_y = g$
Freely Falling Object ≡ thrown vertically up
v_i
θ_i
Maximum height
$v_{ix} = v_i cos\theta_i$=constant
$a_x = 0$
$x_f = x_i + (v_i cos\theta_i)\ t$
θ_i
Range
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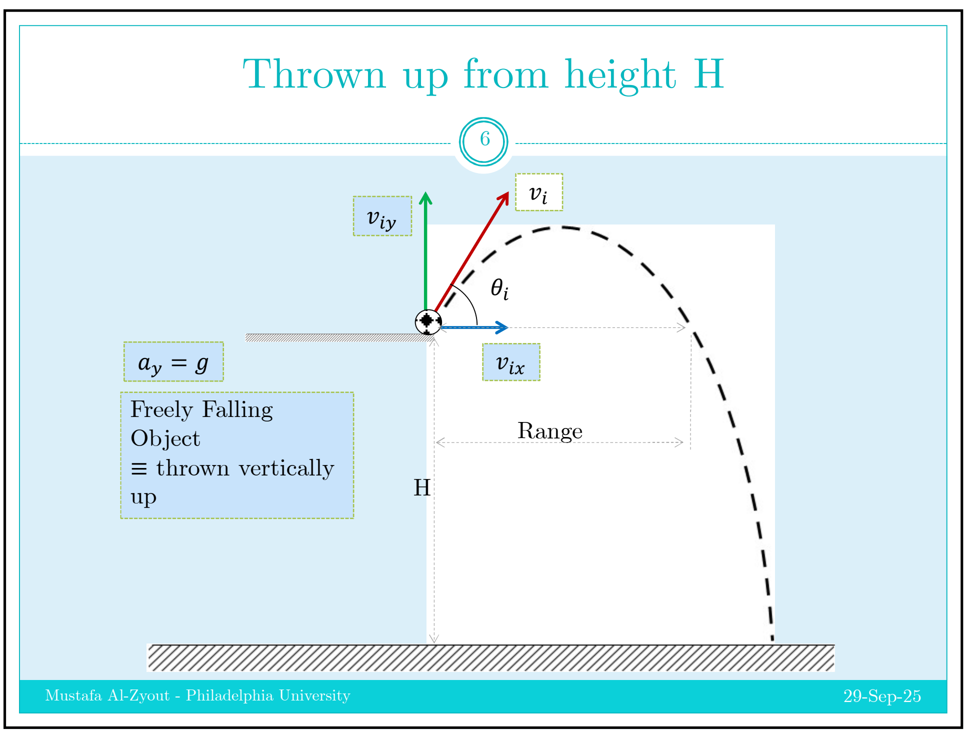
5


Thrown up from height H
6
v_{iy}
v_i
θ_i
$a_y = g$
v_{ix}
Freely Falling Object ≡ thrown vertically up
Range
H
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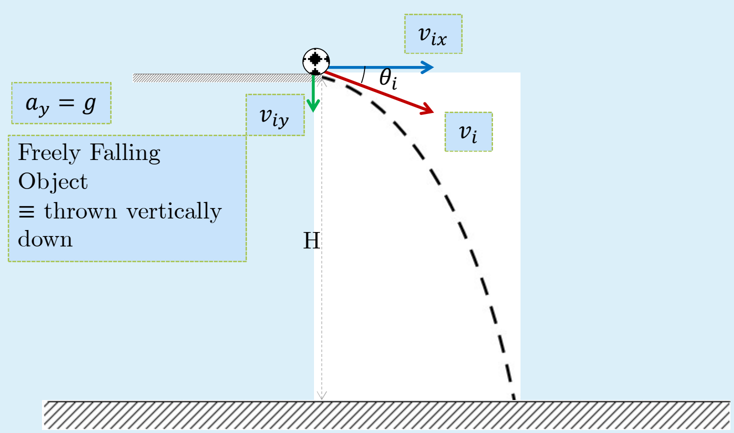
6

Thrown down from height H

7



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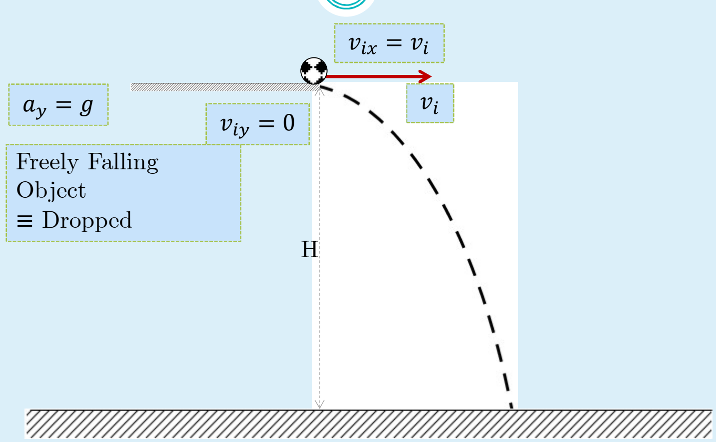
7

Thrown horizontally from height H

8



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Kinematic Equations

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$$\vec{a} = \vec{g} = -9.8\hat{j}(m/s^2)$$

Equations	Missing
$\vec{v}_f = \vec{v}_i + \vec{a}t$	$\Delta\vec{r}$: displacement (m)
$\Delta\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	$\vec{v}_f$: final velocity (m/s)
$\Delta\vec{r} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$	$\vec{v}_i$: initial velocity (m/s)

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Kinematic Equations: Vertical components

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$$g = 9.8\ m/s^2$$

Equations	Missing
$v_{fy} = (v_i sin\theta_i) - gt$	Δy : displacement (m)
$v_{fy}^2 = (v_i sin\theta_i)^2 - 2g\Delta y$	t : time (s)
$\Delta y = (v_i sin\theta_i)t - \frac{1}{2}gt^2$	v_f : final velocity (m/s)
$\Delta y = v_{fy}t + \frac{1}{2}gt^2$	v_i : initial velocity (m/s)

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Kinematic Equations:

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Horizontal components: it is a motion with constant velocity:

- $x_f = x_i + (v_i cos\theta_i)\ t$

Range: is the maximum horizontal distance:

- $R = \frac{v_i^2 sin2\theta_i}{g}$

Maximum height: is the maximum vertical distance:

- $h = \frac{v_i^2 sin^2\theta_i}{2g}$

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Notes

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The horizontal and vertical components of a projectile's motion are completely independent of each other.

Time is the common variable for both components:

- $t_x = t_y = t$

The velocity at the maximum height equals the horizontal component:

- $v_{max.h} = v_{ix} = v_i cos\theta$

The acceleration anywhere along the trajectory is:

- $\vec{a} = \vec{g} = -9.8\hat{\jmath}(m/s^2)$

The path of a projectile is a parabola.

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Notes, …

The maximum range occurs at:

- $\theta_i = 45^\circ$.

Complementary angles will produce the same range.

- The maximum height will be different for the two angles.
- The times of the flight will be different for the two angles.

Height and Range

Saturday, 30 January, 2021 12:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A long jumper leaves the ground at an angle of (20°) above the horizontal and at a speed of (11 m/s).

(A) How far does he jump in the horizontal direction?

(B) What is the maximum height reached?

SOLUTION

to find the range of the jumper:

$$R = \frac{v_i^2 \sin 2\,\theta_i}{g} = \frac{(11.0m/s)^2 \sin 2\,(20.0°)}{9.80m/s^2} = 7.94m$$

SOLUTION

the maximum height reached

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g} = \frac{(11.0m/s)^2 (\sin 2\,0.0)^2}{2(9.80m/s^2)} = 0.722m$$

That's Quite an Arm!

Saturday, 30 January, 2021 12:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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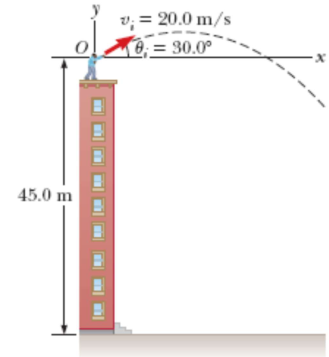
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A stone is thrown from the top of a building upward at an angle of 30° to the horizontal with an initial speed of $20\ m/s$ as shown. The height from which the stone is thrown is $45\ m$ above the ground.

(A) How long does it take the stone to reach the ground?

(B) What is the speed of the stone just before it strikes the ground?



SOLUTION

We have the information $x_i = y_i = 0$, $y_f = -45.0m, a_y = -g$, and $v_i = 20.0m/s$ (the numerical value of y_f is negative because we have chosen the point of the throw as the origin).

Find the initial x and y components of the stone's velocity:

$$v_{xi} = v_i \cos\theta_i = (20.0m/s) \cos 3\,0.0° = 17.3m/s$$

$$v_{yi} = v_i \sin\theta_i = (20.0m/s) \sin 3\,0.0° = 10.0m/s$$

Express the vertical position of the stone from the vertical component

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Substitute numerical values:

$$-45.0m = 0 + (10.0m/s)t + \frac{1}{2}(-9.80m/s^2)t^2$$

Solve the quadratic equation for t:

$$t = 4.22s$$

SOLUTION

To obtain the y component of the velocity of the stone just before it strikes the ground: $v_{yf} = v_{yi} + a_y t$

Substitute numerical values, using $t = 4.22s$:

$$v_{yf} = 10.0m/s + (-9.80m/s^2)(4.22s) = -31.3m/s$$

Use this component with the horizontal component $v_{xf} = v_{xi} = 17.3m/s$to find the speed of the stone at $t = 4.22s$:

xf yf

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3m/s)^2 + (-31.3m/s)^2} = 35.8m/s$$

Is it reasonable that the y component of the final velocity is negative?